Quantum graphs with a group action

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Question: Assume a group G acts on a graph Γ. What does that tell us about Γ?

Theorem (Gross–Tucker, 1987)
A finite directed graph Γ admits a free action of a finite group G iff Γ is isomorphic to the **derived graph** of a **voltage graph**.

Definition
A *voltage graph* is a graph $\tilde{\Gamma}$ together with a labeling of its edges by elements of a finite group G.

Its *derived graph* $\tilde{\Gamma}^\lambda$ is a graph with vertex set $V_{\tilde{\Gamma}} \times G$ with edges

$$(e, g) : (v, g) \longrightarrow (w, \lambda(e)g)$$

for every edge $e : v \longrightarrow w$ in $\tilde{\Gamma}$ and $g \in G$.

Application to operator algebras:

Theorem (Kumjian-Pask, 2000)
For the associated graph C*-algebras we have

$$C^*(\tilde{\Gamma}^\lambda) \cong C^*(\tilde{\Gamma}) \rtimes \hat{G}.$$

Do we find similar results in the non-commutative setting?

Then *graphs* are replaced by *quantum graphs* on *finite quantum sets*.

A finite quantum set is essentially a f.d. C*-algebra B, and a quantum graph is given by a quantum adjacency map $A : B \to B$.

Theorem (S.–Tobolski, 2026)
A quantum graph A on a finite quantum set B admits a **dual action** α of a finite abelian group G iff A is isomorphic to the **derived quantum graph** of a **quantum voltage graph**.

Here, we call α a *dual action* if

$$(B, \alpha) \cong (\tilde{B} \rtimes_\beta \hat{G}, \hat{\beta}).$$

A *quantum voltage graph* is given by a map

$$(A, \lambda) : B \to B \otimes C(\hat{G}),$$

and the *derived quantum graph* is defined on the vertex quantum set

$$\tilde{B} := B \rtimes_\beta \hat{G}$$

We suggestively denote the derived quantum graph by

$$\tilde{A} := (A, \lambda) \rtimes_\beta \hat{G}.$$

Outlook: Is

$$C^*((A, \lambda) \rtimes_\beta \hat{G}) \cong C^*(A) \rtimes G$$

for the associated quantum Cuntz–Krieger C*-algebras?

Thank you!